

**INTERNATIONAL BACCALAUREATE****MATHEMATICS**

Higher Level

Wednesday 6 November 1996 (afternoon)

Paper 1

2 hours

This examination paper consists of 20 questions.

The maximum mark for each question is 4.

The maximum mark for this paper is 80.

This examination paper consists of 13 pages.

INSTRUCTIONS TO CANDIDATES

Write your candidate reference
number in this box:

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DO NOT open this examination paper until instructed to do so.

Answer **ALL** questions in the spaces provided.

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.

EXAMINATION MATERIALS

Required/Essential:

IB Statistical Table
Electronic calculator
Ruler and compasses

Allowed/Optional:

A simple translating dictionary for candidates not working in their own language
Millimetre square graph paper

FORMULAE

Trigonometrical identities:

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = 2 \sin \frac{\alpha + \beta}{2} \sin \frac{\beta - \alpha}{2}$$

$$\cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta = \cos^2 \theta - \sin^2 \theta$$

$$\text{If } \tan \frac{\theta}{2} = t \text{ then } \sin \theta = \frac{2t}{1+t^2} \text{ and } \cos \theta = \frac{1-t^2}{1+t^2}$$

Integration by parts:

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Standard integrals:

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + c \quad (|x| < a)$$

Statistics: If (x_1, x_2, \dots, x_n) occur with frequencies (f_1, f_2, \dots, f_n) then the mean m and standard deviation s are given by

$$m = \frac{\sum f_i x_i}{\sum f_i}, \quad s = \sqrt{\frac{\sum f_i (x_i - m)^2}{\sum f_i}}, \quad i = 1, 2, \dots, n$$

Binomial distribution:

$$p_x = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$$

Maximum marks will be given for correct answers. Where an answer is wrong some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary, or on extra sheets of paper provided these are securely fastened to the cover sheet together with this examination paper.

1. Find all the x values for which each one of the following functions is **not** defined.

(a) $f(x) = \frac{|x-1|}{x}$;

(b) $g(x) = |x-1| - x$;

(c) $h(x) = \sqrt{x^2 - 1}$;

(d) $j(x) = \ln(1 - x^2)$.

Working:

Answers:

(a) _____

(b) _____

(c) _____

(d) _____

2. The end of the minute hand of a railway station clock is 1.2 metres from the centre of the clock. If the end of the hour hand of the same clock from the centre is 0.8 metres, then calculate the speeds, in metres per minute, of the ends of the minute hand and the hour hand.

Working:

Answers:

3. Let

$$A = \begin{pmatrix} 4 & 2 & k \\ 6 & 0 & -1 \end{pmatrix}; \quad B = \begin{pmatrix} 0 & k \\ 2 & 0 \\ -2k & 6 \end{pmatrix}; \quad C = \begin{pmatrix} 1 & 5 \\ 1 & 0 \end{pmatrix}.$$

(a) Find AB .

(b) Find all values of k so that $AB - 2C = 0$.

Working:

Answers:

(a) _____

(b) _____

4. Let $z_1 = 2 + i\sqrt{3}$ and $z_2 = 1 - i$. Find

(a) $z_1 z_2$ and $\frac{z_1}{z_2}$, in the form $a + ib$, where $a, b \in \mathbb{R}$;

(b) $|z_1 z_2|$.

Working:

Answers:

(a) _____

(b) _____

5. Find the sum of all the integers between 200 and 500 which are divisible by 7.

Working:

Answer:

6. Find $\int e^x \sin x \, dx$.

Working:

Answer:

7. A spherical snowball is melting and its volume is decreasing at the rate of 40 cm^3 per minute. Find, in terms of π , the rate of change of its radius when the radius is 5 cm.

Working:

Answer:

8. Find all angles θ , $0^\circ \leq \theta \leq 360^\circ$, correct to the nearest degree, such that

$$3 \sin \theta + 4 \cos \theta = 5.$$

Working:

Answer:

9. A continuous probability density function is described as follows:

$$f(x) = \begin{cases} 0, & x < 0; \\ 4 - 8x; & 0 \leq x \leq k; \\ 0, & x > k. \end{cases}$$

Find

- (a) the value of k ;
- (b) the mean of the distribution.

Working:

Answers:

(a) _____

(b) _____

10. Solve

$$\frac{dy}{dx} + y \tan x = \cos x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

Working:

Answer:

11. A jogger starts running from a point A at a constant speed of 1 km every 3 minutes. He runs for 30 minutes in the direction 70° east of north, then for 15 minutes in the direction 20° west of south to reach a point B . Find the distance AB to the nearest tenth of a kilometre.

Working:

Answer:

12. How many different arrangements, each consisting of 5 different digits, can be formed from the digits 1, 2, 3, 4, 5, 6, 7, if
- (a) each arrangement begins and ends with an even digit?
 - (b) in each arrangement odd and even digits alternate?

Working:

Answers:

- (a) _____
- (b) _____

13. An employer randomly selects three new employees from ten applicants, consisting of six men and four women.
- (a) Find the probability that no women are selected.
 - (b) Find the probability that 2 women and 1 man are selected.

Working:

Answers:

- (a) _____
- (b) _____

14. Smith & Co. produces jars of jam in such a way that the net weight of a jar of jam is normally distributed with mean 450 g, and standard deviation 5 g. What percentage of jars has a net weight of less than 442 g?

Working:

Answer:

15. Express

$$\frac{11x^2 - 9x + 19}{(x - 1)(3x^2 + 4)}$$

as the sum of two partial fractions.

Working:

Answer:

16. A curve is defined by

$$x = (2t - 1), y = \frac{2t}{(t - 1)}.$$

Find the equation of the tangent to the curve when $x = 5$.

Working:

Answer:

17. Find the points on the curve $y^2 - x^2 = 4$ which are closest to the point $(2, 0)$.

Working:

Answers:

18. Find $\int \frac{dx}{x^2 + 6x + 13}$.

Working:

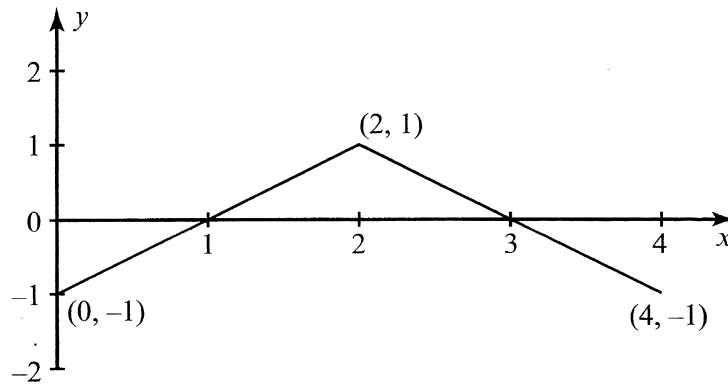
Answer:

19. Find all values of x for which $|x + 3| > 2|x + 1|$.

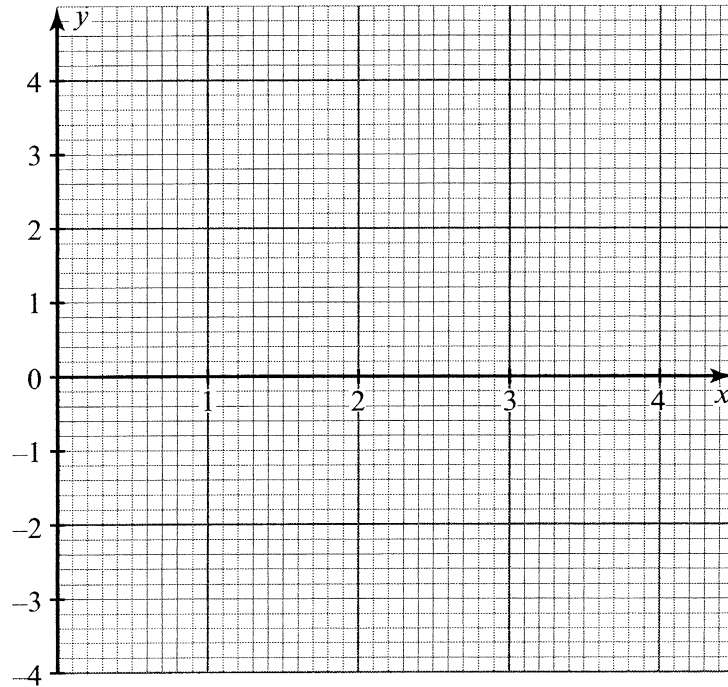
Working:

Answer:

20. The graph of $f(x)$ is given below.



- (a) Draw the graph of $|f(x)|$ on the set of axes below.
- (b) Find the y -intercept of $\frac{1}{f(x)}$.
- (c) Draw the graph of $\frac{1}{f(x)}$ on the same set of axes below.



Answer:

(b) _____